

In this PS' lies on the opposite side TP-T' from PS, such that  $\angle TPS = \angle TPS'$  and  $S'P = SP = 2a$  so that

$$S'P - SP = 2a$$

$$S'P = R + 2a = \frac{V^2 R^2}{V^2 R - 2\mu}$$

The path can then be constructed, since S' is the second focus.

### Kepler's Laws.

The astronomer Kepler, after many years of patient labour, discovered three laws connecting the motions of the various planets about the sun.

They are

1. Each planet describes an ellipse having the sun in one of its foci.
2. The areas described by the radii drawn from the planets to the sun are in the same orbit, proportional to the times of describing them.
3. The squares of the periodic times of the various of the planets are proportional to the cubes of the major axes of their orbits.

From the second law we conclude that the acceleration of each planet, and therefore the force on it is directed towards the sun.

From the first Law it follows that the acceleration of each planet varies inversally as the square of the distance from the sun.

From the third Law it follows

$$T^2 = \frac{4\pi^2}{\mu} a^2$$

The absolute acceleration  $\mu$  (ie the acceleration of unit distance from the sun) is the same for all planets.

Laws simillar to those of Kepler have been found to hold for the planets and their satellites.

It follows foregoing considerations that we may assume Newton's Law of gravitation to be true throughout the solar system.

Kepler's Laws obtained by him by a process of continuously trying hypothesis until he found one that was suitable, he started ~~that~~ with the observations made and recorded for many years by Tycho Brahe a Dane who lived from

A.D. 1546 to 160.

The first and second laws were enunciated by Kepler in 1609 in his book on the motion of the planet Mars. The third Law was announced ten years later in a book entitled on the Harmonies of the world. The explanation of these laws was given by Newton in his Principia published in the year 1687.

Kepler's third Law, is only true on the supposition that the sun is fixed, or that the ~~the~~ Sun is fixed, or that the mass of the planet is neglected in comparison with that of the sun.

A more accurate form is obtained in the following manner.

Let ~~is~~  $S$  be the mass of the Sun,  $P$  that of its planet is neglected in comparison with that of the sun. & the constant of gravitation. The force of attraction between the two is thus  $\gamma \cdot \frac{S \cdot P}{r^2}$

where  $r$  is the distance between the sun and planet at any instant.

The acceleration of the planet is then  $\alpha (= \frac{r S}{r^2})$  towards the Sun. and that of the sun is  $\beta (= \frac{r P}{r^2})$

towards the planet.



To obtain the acceleration of the planet relative to the Sun we must give to both an acceleration  $\beta$  along the line PS. The acceleration of the sun is then zero, and that of the planet is  $\alpha + \beta$  along PS. If in addition we give to each a velocity equal and opposite to that of the Sun we have the motion of P relative to the Sun supposed to be at rest.

The relative acceleration of the planet with respect to the sun then

$$\alpha + \beta = \frac{r(s+p)}{s^2}$$

Hence  $\mu = r(s+p)$  and then we have

$$T = \frac{2\pi}{\sqrt{r(s+p)}} a_1^{3/2} \quad \therefore \frac{s+p}{s+p_1} \cdot \frac{T^2}{T_1^2} = \frac{a^3}{a_1^3}$$

Since Kepler's Law, that  $\frac{T^2}{T_1^2}$  varies as  $\frac{a^3}{a_1^3}$  is very approximately true, it

follows that  $\frac{s+p_1}{s+p}$  is very nearly unity.

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